1. Express $3x^2 - 12x + 5$ in the form $a(x - b)^2 - c$. Hence state the minimum value of y on the curve $y = 3x^2 - 12x + 5$.

[5]

2. Cheung wishes to model the fuel consumption of a car. He tries the quadratic model

$$y = a\,(v - b)^2 + c$$

where y is the fuel needed in litres per 100 km and v is the speed in km h^{-1}.

Travelling as a passenger, he notices that the minimum fuel consumption displayed on the dashboard is 10 litres per 100 km and occurs at 80 km h^{-1}.

(a) Write down the values of b and c for which Cheung's model fits this information. [2]

At 30 km h^{-1} the fuel consumption displayed is 12 litres per 100 km.

(b) Find the value of a for which Cheung's model fits this information. [2]

(c) Use this model to predict the fuel consumption at 110 km h^{-1}. [2]

(d) Sketch the graph of the model for speeds between 30 km h^{-1} and 110 km h^{-1}. [2]

Later in the journey, Cheung notices that fuel consumption of 12 litres per 100 km is displayed at 30 km h^{-1} and also at 110 km h^{-1}. The minimum fuel consumption still occurs at 80 km h^{-1}.

(e) Give a reason why a quadratic model cannot fit all the information Cheung has found. [1]

3. **In this question you must show detailed reasoning.**

Fig. 2 shows the line $2x + y = 6$. The line crosses the x-axis at A and the y-axis at B.

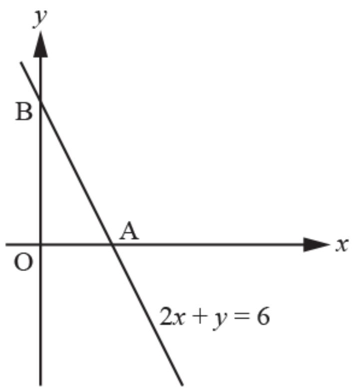


Fig. 2

Find the equation of the quadratic curve which touches the x-axis at A and passes through B. [5]

4. (i) Express $y = x^2 + x + 3$ in the form $y = (x + m)^2 + p$ and hence explain why the curve $y = x^2 + x + 3$ does not intersect the x-axis. [4]

(ii) Find the coordinates of the points of intersection of the curves $y = x^2 + x + 3$ and $y = 2x^2 - 3x - 9$. [4]

(iii) Find the set of values of k for which the curves $y = x^2 + x + k$ and $y = 2x^2 - 3x - 9$ do **not** intersect. [4]

5. The equation of a curve is $y = \dfrac{a}{(x+b)^2}$. Fig. 5 shows the curve for particular values of the constants a and b.

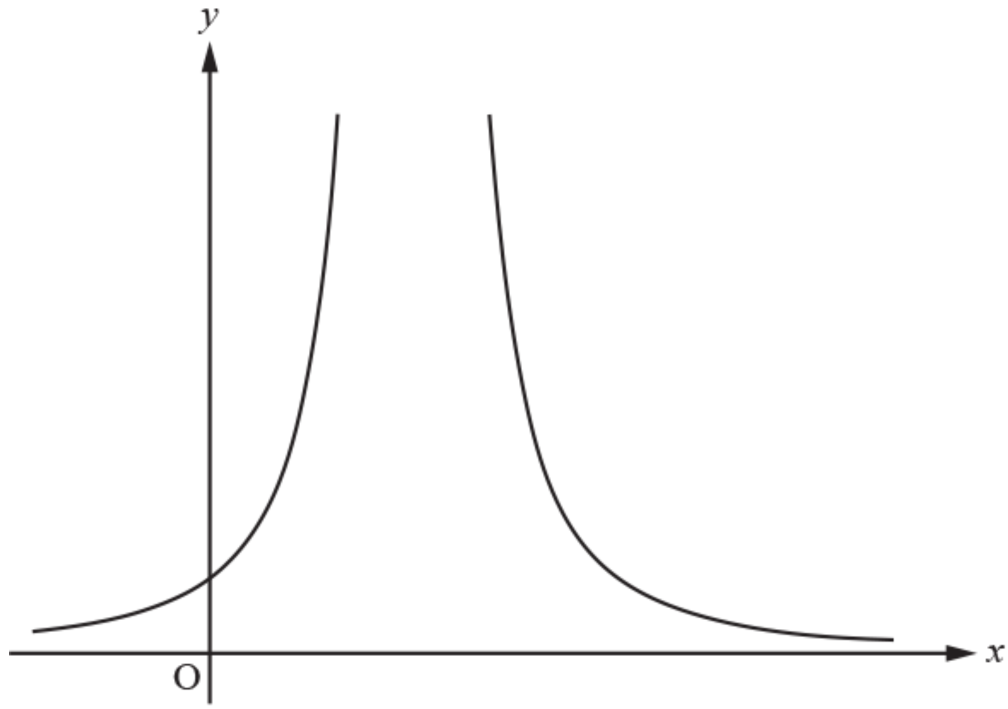


Fig. 5

(a) Write down the equations of the asymptotes, in terms of a and/or b where necessary. [2]

(b) Joe says "For all values of a and b, the curve lies above the x-axis." Determine whether Joe's statement is true or false. [2]

(c) The curve goes through the points (1, 3) and (3, 3). Find the values of a and b. [4]

6.

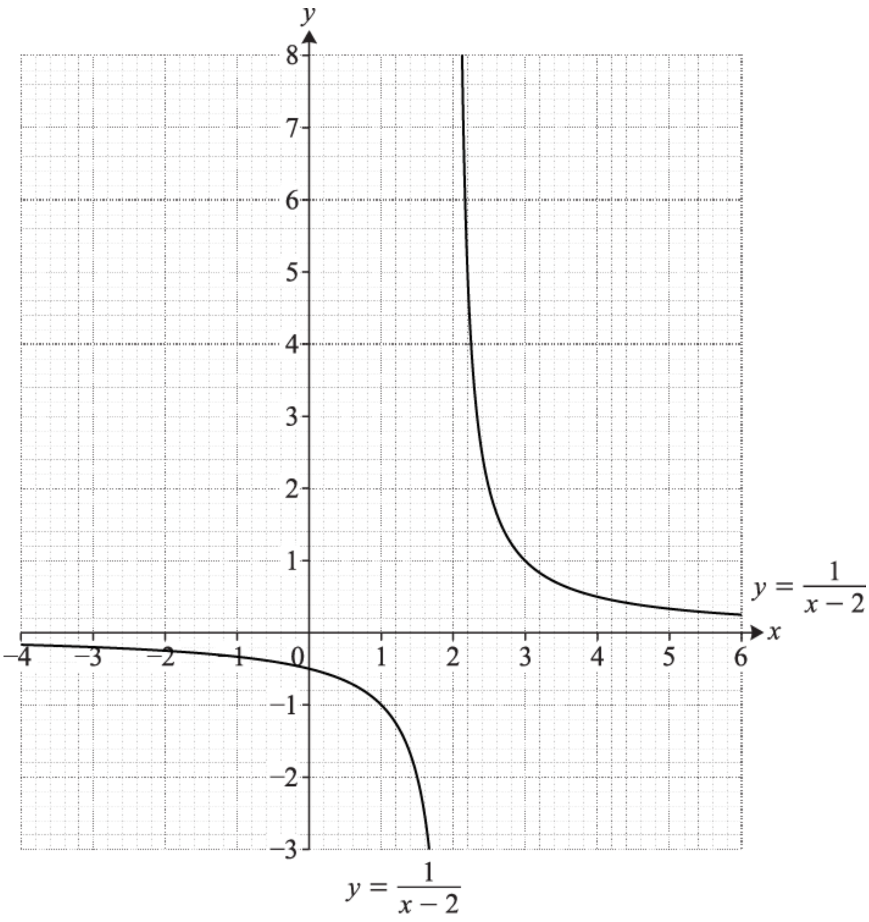


Fig. 12

Fig. 12 shows the graph of $y = \frac{1}{x-2}$.

i. Draw accurately the graph of $y = 2x + 3$ on the copy of Fig. 12 and use it to estimate the coordinates of the points of intersection of $y = \frac{1}{x-2}$ and $y = 2x + 3$.

[3]

ii. Show algebraically that the x-coordinates of the points of intersection of $y = \frac{1}{x-2}$ and $y = 2x + 3$ satisfy the equation $2x^2 - x - 7 = 0$. Hence find the exact values of the x-coordinates of the points of intersection.

[5]

iii. Find the quadratic equation satisfied by the x-coordinates of the points of intersection of $y = \frac{1}{x-2}$ and $y = -x + k$. Hence find the exact values of k for which $y = -x + k$ is a tangent to $y = \frac{1}{x-2}$.

[4]

7. You are given that $f(x) = (x + 2)^2(x - 3)$.

i. Sketch the graph of $y = f(x)$.

[3]

ii. State the values of x which satisfy $f(x + 3) = 0$.

[2]

8. You are given that $f(x) = (2x - 3)(x + 2)(x + 4)$.

i. Sketch the graph of $y = f(x)$.

[3]

ii. State the roots of $f(x - 2) = 0$.

[2]

iii. You are also given that $g(x) = f(x) + 15$.

A. Show that $g(x) = 2x^3 + 9x^2 - 2x - 9$.

[2]

B. Show that $g(1) = 0$ and hence factorise $g(x)$ completely.

[5]

9.

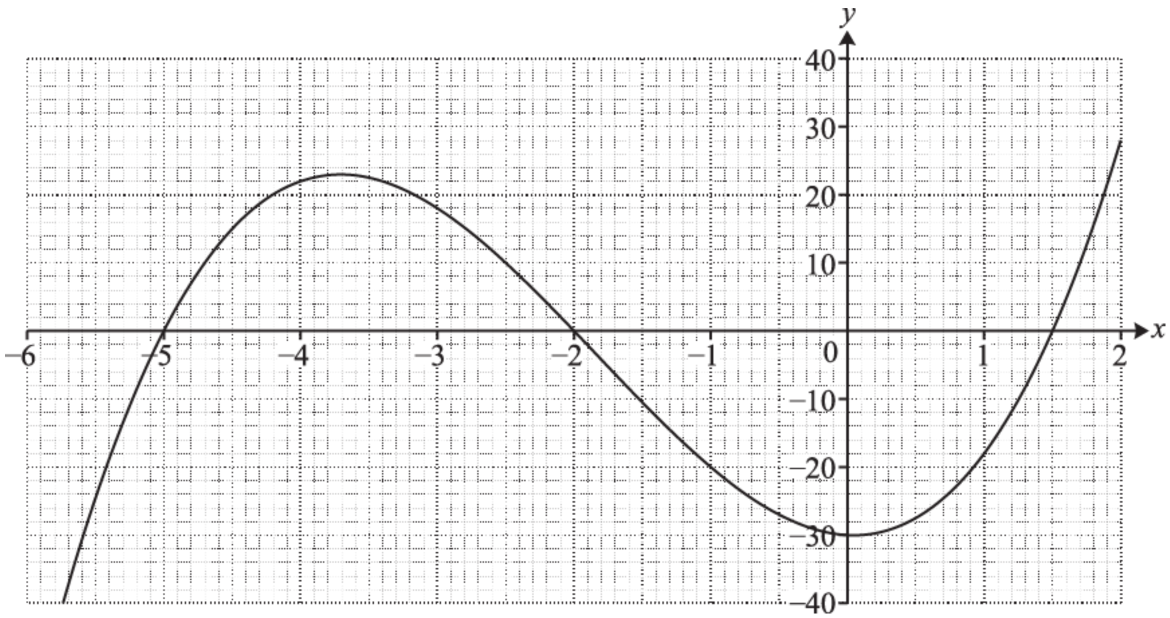


Fig. 12

Fig. 12 shows the graph of a cubic curve. It intersects the axes at (–5, 0), (–2, 0), (1.5, 0) and (0, –30).

i. Use the intersections with both axes to express the equation of the curve in a factorised form.

[2]

ii. Hence show that the equation of the curve may be written as $y = 2x^2 + 11x^2 - x - 30$.

[2]

iii. Draw the line $y = 5x + 10$ accurately on the graph. The curve and this line intersect at (–2, 0); find graphically the x-coordinates of the other points of intersection.

[3]

iv. Show algebraically that the x-coordinates of the other points of intersection satisfy the equation

$$2x^2 + 7x - 20 = 0.$$

Hence find the exact values of the x-coordinates of the other points of intersection.

[5]

10. i. Find the coordinates of the points of intersection of the curve $y = 2x^2 - 5x - 3$ with the axes.

[3]

ii. Find the coordinates of the points of intersection of the curve $y = 2x^2 - 5x - 3$ and the line $y = x + 3$.

[4]

iii. Find the set of values of k for which the line $y = x + k$ does not intersect the curve $y = 2x^2 - 5x - 3$.

[5]

11. i. Solve the equation $(x-2)^2 = 9$.

[2]

ii. Sketch the curve $y = (x - 2)^2 - 9$, showing the coordinates of its intersections with the axes and its turning point.

[3]

12. (a) Express $x^2 + 4x + 7$ in the form $(x + b)^2 + c$. [2]

(b) Explain why the minimum point on the curve $y = (x + b)^2 + c$ occurs when $x = -b$. [1]

13. (See Insert for Practice2 64003.)

[2]

Describe the transformation that maps the curve $y = \mathrm{f}(x)$ onto the curve

(a) $y = \mathrm{f}\left(x - \frac{b}{3a}\right)$.

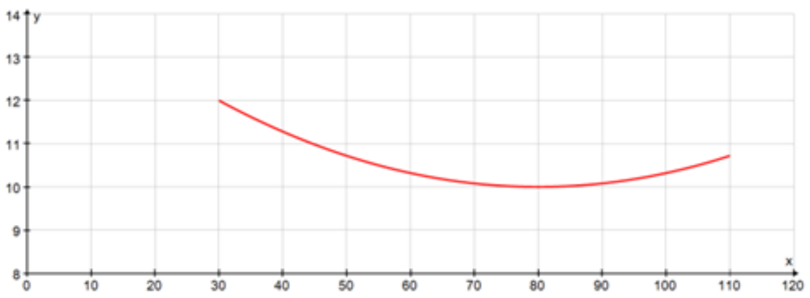
[1]

(b) Given that α is a root of $\mathrm{f}\left(x - \frac{b}{3a}\right) = 0,$ write down a root of $\mathrm{f}(x) = 0$ in terms of α, a and b.

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1			$3(x-2)^2-7$ isw or $a=3$, $b=2$ $c=7$ www	M4.	B1 each for $a=3$, $b=2$ oe and B2 for $c=7$ oe or M1 for $[-]\frac{7}{3}$ or for 5 $-$ *their* a(*their* b)^2 or for $\frac{5}{3}-(\textit{their } b)^2$ soi B0 for (2, −7)	condone omission of square symbol; ignore '= 0' may be implied by their answer
			−7 or ft	B1	**Examiner's Comments** Some who completed the square correctly lost the final mark by giving the minimum point of (2, −7) rather than the minimum y-value. Most common part-correct answers were getting the values of a and b correct but ignoring the multiple of 3 in establishing any value of c. The most common wrong values of b were −6 (dividing the '$-12x$' by 2) and 4 (taking the 3 out as a common factor and forgetting to divide by 2).	may be obtained by starting again eg with calculus
			Total	5		
2		a	$b=80$ $c=10$	B1(AO 1.2) B1(AO 3.3) [2]		

		b	$12 = a(30 - 80)^2 + 10$ $a = \dfrac{2}{50^2} = 0.0008$	M1(AO 3.3) A1(AO 1.1b) [2]	Substituting $v = 30$, $y = 12$ in the model with their values for b and c		
		c	When $v = 110$, $y = 0.0008(110 - 80)^2 + 10$ $y = 10.72$	M1(AO 3.4) A1(AO 1.1b) [2]	Substitution in the model with their values for a, b and c		
		d		G1(AO 1.1a) G1(AO 1.1a) [2]	Curve with minimum point at (80, 10) End-points (30, 12) and (110, 10.72)	Condone graph continuing outside the given interval	
		e	A quadratic model giving the same value at $v = 30$ and $v = 110$ would have to have the minimum point halfway between, at $v = 70$, by symmetry This is not the case so a quadratic model is not suitable	E1(AO 3.5b) [1]	Accept equivalent statement involving symmetry, or any other sensible argument referring to relevant numerical values		
			Total	9			

3			DR Substitute $x = 0$ or $y = 0$ in $2x + y = 6$ Line crosses axes at (0, 6) and (3, 0) Quadratic with factor $(x - 3)$ Repeated factor $(x - 3)$ $y = \frac{2}{3}(x-3)^2$ oe	M1(AO 3.1a) A1(AO 1.1) M1(AO 3.1a) M1(AO 1.1) A1(AO 2.1) [5]	Must follow from clear reasoning		
			Total	5			
4		i	$\left(x+\frac{1}{2}\right)^2 + 2\frac{3}{4}$ oe min $y = 2\frac{3}{4}$ oe or ft, isw or showing that if y = 0, their $\left(x+\frac{1}{2}\right)^2$ is negative, so no real roots [or no solution]	3 B1 [4]	B1 for $m = ½$ oe B2 for $p = 2\frac{3}{4}$ oe or M1 for 3 – their m^2 ft their p, provided $p > 0$; ignore x value of min pt stated, even if wrong ft B0 if only say tp rather than min, though need not justify min	Ignore '=0' M0 if $m = 0$ B0 if explanation not 'hence' eg using $b^2 - 4ac$ on $x^2 + x + 3 = 0$ condone B1 for min pt $= 2\frac{3}{4}$	

					Examiner's Comments Most completed the square correctly. Some candidates did not take notice of the 'hence' in the question and used the discriminant, which did not gain the final mark.	
		ii	$x^2 - 4x - 12$[=0] $(x-6)(x+2)$[=0] $x = 6$ or -2 $y = 45$ or 5	M1 M1 A1 A1 **[4]**	condone one error; for equating and simplifying to solvable form for factors giving at least two terms correct, ft, or for subst in formula with at most one error ft — rearranging to zero not required if they go on to complete the square; similarly for attempt at completing square allow **A1** for coords with x values 6 and -2 but wrong y values or **A1** each for (6, 45) and (-2, 5) **Examiner's Comments** Finding the coordinates of the points of intersection of the two curves was done well. A few forgot to work out both coordinates, and some, having found x to be 6 or -2, put (6, 0) and (-2, 0).	
		iii	$x^2 - 4x - (9 + k)$[=0]	M1	condone one error, but must include k — Eg allow M1 for $y = x^2 - 4x - (9 + k)$ or for $x^2 - 4x - (-9 - k)$[=0]	

					M0 if not all on one side of equation, unless completing the square	for completing the square method: eg $(x-2)^2 - 4 = 9 + k$ earns M1 and the second A1, which is likely to be earned before the first A1
			$b^2 - 4ac <$ oe soi	A1	may be earned near end with correct inequality sign used there	allow '$b^2 - 4ac$ is negative' oe; 0 for just 'discriminant < 0' unless implied by later work; A0 for $\sqrt{b^2 - 4ac} < 0$; for completing the square, allow A1 for $4 + 9 + k < 0$ oe
			$16 + 4 \times (9 + k)$ (<0) oe or ft	A1	for correct substitution ft into $b2 - 4ac$; brackets / signs must be correct; allow recovery from eg wrong bracket earlier but	can be earned with equality or wrong inequality, or in formula (ignore rest of formula); for completing the square method, will see $4 + 9 + k$ oe, or on opposite sides of an inequality / equation eg $9 + k < -4$ M0 for trials of values of k in $b^2 - 4ac$

			$k < -13$ www	A1 [4]	A0 if recovery line contains additional error(s) A0 for just $k < -52/4$ **Examiner's Comments** This question posed problems for some candidates who were unsure of an appropriate strategy. However, a good number of the candidates found the set of values of k successfully. A significant minority, while knowing that they needed to use the discriminant, made arithmetical and/or algebraic errors, often caused by poor use of brackets. Few used the completing the square method.	
			Total	12		
5		a	$x = -b$ $y = 0$	B1(AO2.2a) B1(AO1.1) [2]		

		b	For negative a, $\frac{a}{(x+b)^2} < 0$ So in this case the curve lies below the x-axis and Joe's statement is false	B1(AO2.3) E1(AO2.4) [2]	If zero scored, SC1 for $(x+b)^2$ is never negative oe	
		c	$x = -b$ is a line of symmetry $b = -2$ $3 = \frac{a}{(1+b)^2}$ a = 3	M1(AO3.1a) A1(AO2.2a) M1(AO1.1a) A1(AO1.1) [4]	Or $x = 2$ is a line of symmetry Or $3 = \frac{a}{(3+b)^2}$ Or $(1+b)^2 = (3+b)^3$	
			Total	8		
6		i	$y = 2x + 3$ drawn accurately	M1	at least as far as intersecting curve twice	ruled straight line and within 2mm of (2, 7) and (–1, 1)
		i	(–1.6 to –1.7, –0.2 to –0.3)	B1	intersections may be in form $x = ..., y = ...$	
		i	(2.1 to 2.2, 7.2 to 7.4)	B1		if marking by parts and you see work relevant to (ii), put a yellow line here and in (ii) to alert you to look
		i	**Revised tolerances for modified papers for visually impaired candidates (graph in (i) with 6mm squares)** $y = 2x + 3$ drawn accurately	M1	at least as far as intersecting curve twice	ruled straight line and within **3** mm of (2, 7) and (–1, 1)
		i	(–1.6 to –1.**8**, –0.2 to –0.3)	B1	intersections may be in form $x = ..., y = ...$	

		i	(2.1 to 2.**3**, 7.**1** to 7.4)	B1		if marking by parts see work relevant to (ii), put a yellow line here and in (ii) to alert you to look
		i			Examiner's Comments Almost all candidates were able to draw the line accurately. Omission of one or both of the signs on the negative intersections was quite common; a few reversed the coordinates. A few just wrote the two x-values only.	
		ii	$\dfrac{1}{x-2}=2x+3$	M1	or attempt at elimination of *x* by rearrangement and substitution	may be seen in (i) – allow marks; the part (i) work appears at the foot of the image for (ii) so show marks there rather than in (i)
		ii	$1 = (2x + 3)(x - 2)$	M1	condone lack of brackets	implies first M1 if that step not seen
		ii	$1 = 2x^2 - x - 6$ oe	A1	for correct expansion; need not be simplified; NB A0 for $2x^2 - x - 7 = 0$ without expansion seen [given answer]	implies second M1 if that step not seen after $\dfrac{1}{x-2}=2x+3$ seen
		ii	$\dfrac{1\pm\sqrt{1^2-4\times2\times-7}}{2\times2}$ oe $\dfrac{1\pm\sqrt{57}}{4}$ isw	M1	use of formula or completing square on given equation, with at most one error	completing square attempt must reach at least $[2](x - a)^2 = b$ or $(2x - c)^2 = d$ stage oe with at most one error
		ii		A1	isw e.g. coordinates; after completing square, accept $\dfrac{1}{4}\pm\sqrt{\dfrac{57}{16}}$ or better	

					Examiner's Comments Most were able to obtain the correct equation and many went on to solve it successfully, although as expected, there were some errors in using the formula, especially frequently in evaluating the discriminant after correct substitution.	
		iii	$\frac{1}{x-2}=-x+k$ and attempt at rearrangement	M1		
		iii	$x^2-(k+2)x+2k+1[=0]$	M1	for simplifying and rearranging to zero; condone one error; collection of x terms with bracket not required	e.g. M1 bod for $x^2-(k+2)x+2k$ or M1 for $x^2-2kx+2k+1[=0]$
		iii	$b^2-4ac=0$ oe seen or used	M1		$=0$ may not be seen, but may be implied by their final values of k
		iii	$[k=]$ 0 or 4 as final answer, both required	A1	SC1 for 0 and 4 found if 3rd M1 not earned (may or may not have earned first two Ms) **Examiner's Comments** After the previous part, most candidates realised that they had to equate the two expressions and manipulate the resulting equation, although many had problems dealing with the 'k' terms ('$kx+2x=2kx$' for instance). Most candidates stopped there, but some realised that they needed to use '$b^2-4ac=0$' to establish the final values of k. Some were confused with the k and x terms and were unable to identify the coefficients correctly or made errors in simplifying the equation. A few candidates used their graphs to establish the results for k. A few tried to apply calculus but rarely with any success.	e.g. obtained graphically or using calculus and / or final answer given as a range
			Total	12		

7		i	sketch of cubic the right way up, with two tps	B1		No section to be ruled; no curving back; condone some curving out at ends but not approaching another turning point; condone some doubling (eg erased curves may continue to show); ignore position of turning points for this mark
		i	their graph touching the x-axis at -2 and crossing it at 3 and no other places	B1	if intns are not labelled, they must be shown nearby	mark intent if 'daylight' between curve and axis at $x = -2$
		i	intersection of y-axis at -12	B1		if no graph but -12 marked on y-axis, or in table, allow this 3rd mark
		ii	-5 and 0	B2	B1 each; allow B2 for -5, -5, 0; or B1 for both correct with one extra value or for $(-5, 0)$ and $(0, 0)$ or SC1 for both of 1 and 6 **Examiner's Comments** Most candidates obtained full marks for sketching the cubic curve, although their cubics were often unshapely, partly due to the incorrect assumption by many that there was a turning point where the graph crossed the y-axis. Most had the cubic the correct way up and realised that it touched the x-axis at -2. A few labelled the y-intersection as 12 rather than -12. A minority sketched parabolas. Full marks were less common in the second part; a small proportion translated to the right rather than to the left as $f(x+3)$ required. A larger minority did not know what to do and obtained no marks, often giving the single root $x = -3$.	if their graph wrong, allow -5 and 0 from starting again with eqn, or ft their graph with two intns with x-axis
			Total	5		

8		i	sketch of cubic the right way up, with two tps and clearly crossing the *x* axis in 3 places	B1		no section to be ruled; no curving back; condone slight 'flicking out' at ends but not approaching another turning point; condone some doubling (eg erased curves may continue to show); accept min tp on *y*-axis or in 3rd or 4th quadrant; curve must clearly extend beyond the *x* axis at both 'ends'
		i	crossing / reaching the *x*-axis at −4, −2 and 1.5	B1	intersections must be shown correctly labelled or worked out nearby; mark intent	accept curve crossing axis halfway between 1 and 2 if 3/2 not marked
		i	intersection of *y*-axis at −24	B1		NB to find −24 some are expanding f(*x*) here, which gains M1 in iiiA. If this is done, put a yellow line here and by (iii)A to alert you; this image appears again there
		i			**Examiner's Comments** Most candidates were able to sketch the correct shape for the cubic (the correct way up) and the majority were also able to correctly label the interceptions on the x-axis, although some gave the positive x intercept as ½ or 2/3 or 3. A few candidates failed to label the y-intercept or gave a wrong value such as 12 or -12. Some candidates drew their graph stopping at one of the roots (usually when x = – 4) instead of crossing the x-axis. Only a small number of candidates drew the graph upside-down and a handful drew the wrong shape altogether.	
		ii	−2, 0 and 7/2 oe isw or ft their intersections	2	B1 for 2 correct or ft or for (−2, 0) (0, 0) and (3.5, 0) or M1 for $(x + 2)\,x\,(2x − 7)$ oe or SC1 for −6, −4 and −1/2 oe	

					Examiner's Comments Quite a few errors were seen here, although a minority knew what to do and wrote down the correct values. Some gave factors or coordinates instead of roots, some solved x – 2 = 0 to give x = 2 as the root, and some went back to the equation but made an algebraic error in replacing x with x – 2, reaching $2x – 5$ as a factor instead of $2x – 7$.	
		iii	*(A)* correct expansion of product of 2 brackets of f(x)	M1	need not be simplified; condone lack of brackets for M1 or allow M1 for expansion of all 3 brackets, showing all terms, with at most one error: $2x^3 + 4x^2 + 8x^2 – 3x^2 + 16x – 12x – 6x – 24$	e.g. $2x^2 + 5x – 12$ or $2x^2 + x – 6$ or $x^2 + 6x + 8$ may be seen in (i) – allow the M1; the part (i) work appears at the foot of the image for (iii)A, so mark this rather than in (i)
		iii	correct expansion of quadratic and linear and completion to given answer	A1	for correct completion if all 3 brackets already expanded, with some reference to show why –24 changes to –9 **Examiner's Comments** The first part was generally well done; most correctly expanded two brackets and continued to simplify and add 15 to get the required result. Common errors were: not dealing correctly with the 15 such as saying g(x) = –15 to get the result, and errors in expanding or collecting terms. There was some poor ‘mathematical grammar’ with the ‘+15’ often appearing out of nowhere.	condone lack of brackets if they have gone on to expand correctly; condone ‘+15’ appearing at some stage NB answer given; mark the whole process
		iii	*(B)* g(1) = 2 + 9 – 2 – 9 [=0]	B1	allow this mark for (x – 1) shown to be a factor and a statement that this means that x = 1 is a root [of g(x) = 0] oe	B0 for just $g(1) = 2(1)^3 + 9(1)^2 – 2(1) – 9$ [=0]
		iii	attempt at division by (x – 1) as far as $2x^3 – 2x^2$ in working	M1	or inspection with at least two terms of quadratic factor correct	M0 for division by $x + 1$ after g(1) = 0 unless further working such as g(–1) = 0 shown, but this can go on to gain last M1A1

		iii	correctly obtaining $2x^2 + 11x + 9$	A1	allow B2 for another linear factor found by the factor theorem	NB mixture of met be seen in this part – mark equivalently eg three uses of factor theorem, or two uses plus inspection to get last factor;
		iii	factorising a correct quadratic factor	M1	for factors giving two terms correct; eg allow M1 for factorising $2x^2 + 7x - 9$ after division by $x + 1$ allow $2(x + 9/2)(x + 1)(x - 1)$ oe;	allow M1 for $(x + 1)(x + 18/4)$ oe after −1 and −18/4 oe correctly found by formula
		iii	$(2x + 9)(x + 1)(x - 1)$ isw	A1	dependent on 2nd M1 only; condone omission of first factor found; ignore '= 0' seen **Examiner's Comments** In part (*B*) most candidates correctly showed g(1) = 0 although some failed to show enough working. Candiates were well-versed, in general, with the techniques of long division or inspection so that most achieved the correct quadratic factor and were able to go on and factorise this to gain full marks. Some tried to use the quadratic formula and then only gave $(x + 1)(x + 4.5)$ oe as factors.	SC alternative method for last 4 marks: allow first M1A1 for $(2x + 9)(x^2 - 1)$ and then second M1A1 for full factorisation
			Total	12		
9		i	$y = (x + 5)(x + 2)(2x - 3)$ or $y = 2(x + 5)(x + 2)(x - 3/2)$	2	M1 for $y = (x + 5)(x + 2)(x - 3/2)$ or $(x + 5)(x + 2)(2x - 3)$ with no equation or $(x + 5)(x + 2)(2x - 3) = 0$ but M0 for $y = (x + 5)(x + 2)(2x - 3) - 30$ or $(x + 5)(x + 2)(2x - 3) = 30$ etc **Examiner's Comments** Candidates struggled with this question. Often they managed to produce the product of binomial factors $x + 2$ $x + 5$ $x - 1.5$ and failed to put it equal to y or put it equal to 0. Those who did have the correct product still very often had an expression only or equated to 0. Many candidates thought that the information	allow 'f(x) =' instead of 'y =' ignore further work towards (ii) but do not award marks for (i) in (ii)

					about the y-intercept indicated that they should perform a vertical translation and an answer of $y = x + 2\ x + 5\ x - 1.5 - 30$ was fairly common among weaker students. Some candidates had an epiphany in part (ii) when they realised that their coefficients should be twice the size and sensibly went back to this part and corrected their error.	
		ii	correct expansion of a pair of their linear two-term factors ft isw	M1	ft their factors from (i); need not be simplified; may be seen in a grid	allow only first M1 for expansion if their (i) has an extra −30 etc
		ii	correct expansion of the correct linear and quadratic factors and completion to given answer $y = 2x^3 + 11x^2 - x - 30$	M1	must be working for this step before given answer or for direct expansion of all three factors, allow M2 for $2x^3 + 10x^2 + 4x^2 - 3x^2 + 20x - 15x - 6x - 30$ oe (M1 if one error) or M1M0 for a correct direct expansion of $(x + 5)(x + 2)(x - 3/2)$ condone lack of brackets if used as if they were there **Examiner's Comments** Many scored only one mark in this part, for correctly expanding a pair of their binomial factors, even after making an error in part (i). As said previously, the light dawned for many in this question and it was good to see that some of these made corrections to part (i). However, many did not and very often there would be a multiplication by 2 done at the end — with or without some attempt at justification for it.	do not award 2nd mark if only had $(x - 3/2)$ in (i) and suddenly doubles RHS at this stage condone omission of '$y =$' or inclusion of '= 0' for this second mark (some cands have already lost a mark for that in (i)) allow marks if this work has been done in (i) – mark the copy of (i) that appears below the image for (ii)
		iii	ruled line drawn through (−2, 0) and (0, 10) and long enough to intersect curve at least twice	B1	tolerance half a small square on grid at (−2, 0) and (0, 10)	insert BP on spare copy of graph if not used, to indicate seen – this is included as part of image, so scroll down to see it
		iii	−5.3 to −5.4 and 1.8 to 1.9	B2	B1 for one correct ignore the solution −2 but allow B1 for both values correct but one extra or for wrong 'coordinate' form such as (1.8, −5.3) **Examiner's Comments**	accept in coordinate form ignoring any y coordinates given

					Candidates found this straightforward on the whole, with many scoring full marks for this part. Nearly all drew an accurate line of sufficient length to intersect the curve in three places. Occasionally some read the scale incorrectly when finding the negative solution or were careless with signs, omitting the negative when writing it down.	
		iv	$2x^3 + 11x^2 - x - 30 = 5x + 10$	M1	for equating curve and line; correct eqns only	annotate this question if partially correct
		iv	$2x^3 + 11x^2 - 6x - 40$ [= 0]	M1	for rearrangement to zero, condoning one error	
		iv	division by $(x + 2)$ and correctly obtaining $2x^2 + 7x - 20$	M1	or showing that $(x + 2)(2x^2 + 7x - 20) = 2x^3 + 11x^2 - 6x - 40$, with supporting working	
		iv	substitution into quadratic formula or for completing the square used as far as $\left(x+\frac{7}{4}\right)^2 = \frac{209}{16}$ oe	M1	condone one error eg a used as 1 not 2, or one error in the formula, using given $2x^2 + 7x - 20 = 0$	
		iv	$[x =]\dfrac{-7 \pm \sqrt{209}}{4}$ oe isw	A1	dependent only on 4th M1 **Examiner's Comments** There were many attempts to substitute their answers from part (iii) into the given quadratic. Many candidates did not know how to obtain this quadratic, although most eventually went on to attempt to solve it using the formula, sometimes making arithmetic errors in so doing. Of those who did attempt to derive the quadratic, there were several attempts at equating the wrong pair of equations. Some who started correctly expected to see the given answer immediately and stopped at the simplified cubic they had obtained, sometimes having an erroneous −20. Relatively few were able to show that the quadratic factor was the required one, by tong division or by showing multiplying out. A very few candidates used an elegant method of equating the line and cubic and using the factorised form of each to cancel a factor of $x + 2$ on both sides before simplifying.	
			Total	12		

10		i	(0, −3)	B1	condone $y = -3$, isw	if not coordinates, must be clear which is x and which is y
		i	(− 1/2 , 0) and (3, 0) www	B2	condone $y = -1/2$, and 3; **B1** for one correct www or **M1** for $(2x + 1)(x - 3)$ or correct use of formula or reversed coordinates	**Examiner's Comments** Many candidates earned all 3 marks in this part. Some forgot to find the y-intercept. A few used the quadratic formula or completed the square, perhaps not realising that factorising was possible.
		ii	$2x^2 - 6x - 6$[= 0] isw or $x^2 - 3x - 3$ [= 0] or $2y^2 - 18y + 30$ [=0]	M1	for equating curve and line, and rearrangement to zero, condoning one error	allow rearranging to constant if they go on to attempt completing the square
		ii	use of formula or completing the square, with at most one error	M1	no ft from $2x^2 - 6x = 0$ or other factorisable equations	if completing the square must get to the stage of complete square only on lhs as in 9(ii) A0 for unsimplified y coords eg $\frac{3+\sqrt{21}}{2}+3$
		ii	$\left(\frac{6\pm\sqrt{84}}{4}, \frac{18\pm\sqrt{84}}{4}\right)$ or $\left(\frac{3\pm\sqrt{21}}{2}, \frac{9\pm\sqrt{21}}{2}\right)$ oe isw	A2	**A1** for one set of coords or for x values correct (or ys from quadratic in y); need not be written as coordinates	**Examiner's Comments** The majority of candidates chose the straightforward approach and equated the line and curve given. These candidates most often simplified correctly to a required form and applied the formula. This was often very

						well carried out. A very good number of candidates earned 3 marks using this approach. Some attempted to complete the square. Those who, sensibly, divided through by 2 before doing so were usually successful – those who did not were less successful. Most candidates struggled to find and simplify the *y*-coordinates. Some simply omitted them and the many who attempted them often just wrote the *x* coordinates '+ 3' or failed to convert the 3 being added to a fraction of a common denominator to add to the *x*-coordinate. Some candidates made their solution unnecessarily complicated by rearranging the equation of the line and substituting for *x*. These candidates often omitted to take the solitary *y*-term into account and mostly scored no more than the first two marks.
		iii	$2x^2 - 5x - 3 = x + k$	M1	for equating curve and line	
		iii	$2x^2 - 6x - 3 - k$ [= 0]	M1	for rearrangement to zero, condoning one error, but must include k ;this second M1 implies the first, eg it may be obtained by subtracting the given equations	

iii	$b^2 - 4ac < 0$ oe for non-intersecting lines	M1	eg allow for just quoting this condition; may be earned near end with correct inequality sign used there allow 'discriminant is negative' if further work implies $b^2 - 4ac$	some may use condition for intersecting lines or for a tangent and then swap condition at the end; only award this M1 and the final A mark if the work is completely clear
iii	$36 - 8 \times -(3 + k)$ [< 0] oe	A1	for correct substitution into $b^2 - 4ac$; no ft from wrong equation; if brackets missing or misplaced, must be followed by a correct simplified version	can be earned with equality or wrong inequality, or in formula – this mark is not dependent on the 3rd M mark;
iii	$k < -\frac{15}{2}$ oe		isw if 3rd M1 not earned, allow **B1** for $-\frac{15}{2}$ obtained for k with any symbol	
iii	or, for those using a tangent condition with trials to find the boundary value			mark one mark scheme or another, to the advantage of the candidate, but not a mixture of schemes
iii	rearrangement with correct boundary value of k eg $2x^2 - 6x + 4.5$ [= 0] or $2x^2 - 6x - (3 - 7.5)$ [= 0]	M2	**M1** for $2x^2 - 5x - 3 = x - 7.5$	M0 for trials with wrong values without further progress, though may still earn an M1 for $b^2 - 4ac < 0$
iii	showing $36 - 8 \times -(3 - 7.5) = 0$ or $36 - 8 \times 4.5 = 0$ oe	M1	may be in formula implies previous M2	
iii	$k < -\frac{15}{2}$ oe	A2	**B1** for $-\frac{15}{2}$ obtained for k as final answer with any symbol	

		iii	or, for using tangent with differentiation			
		iii	$y = 4x - 5$	M1		
		iii	[when $y = x + k$ is tgt] $4x - 5 = 1$	M1		
		iii	$x = 1.5$, $y = -6$	A1		
		iii	$-6 = 1.5 + k$ or $k = -7.5$ oe	A1		
		iii	$k < -7.5$ oe	A1		**Examiner's Comments** Some candidates were unable to cope with the constant of the equation they had formed being in terms of k. Many equated the line and curve, as before, and found $2x^2 - 6x - 3 - k = 0$ and then, rather than applying $b^2 - 4ac < 0$, they wrote $2x^2 - 6x - 3 = k$ and tried to apply $b^2 - 4ac < 0$ to the left hand side. Those who did work with $2x^2 - 6x - 3 - k = 0$ were almost always successful. Some candidates made sign errors through carelessness. Some introduced wrong brackets into their equation in an attempt to group the c term, such as $-(3 - k)$. Some candidates correctly substituted into $b^2 - 4ac < 0$ but were unable to multiply out correctly. The result 36 −8

						$-(3+k)$ was not u ncommon amongst these candidates. Other candidates used trials on $2x^2 - 6x - 3 - k = 0$ to find the boundary value ie the constant that gave $b^2 - 4ac = 0$. These often scored 3 marks, but sign errors usually resulted in the loss of the final 2 marks. A very few candidates used a calculus approach. In most cases, once $y' = 4x - 5$ had been found, it was equated to 0 and the minimum point established, thinking that this would be helpful, then no further progress was made. Candidates' setting out in this question was often poor and difficult to make sense of – particularly if they had had several attempts or had used trials. Some candidates lost marks as they restarted several times, with each time being worth less than the previous attempt! Candidates should take care in this regard – and indicate which of their attempts they intend to be taken as the answer in these cases.
			Total	12		

11		i	[x =] 5, [x =] −1 www	2	**M1** for $x - 2 = \pm 3$ or for $(x - 5)(x + 1)$ [=0]	0 for just $x = 5$ or for $x - 2 = 3$
		ii	parabola shape curve the correct way up	1	must extend beyond x-axis;	condone **‘U’** shape or very slight curving back in/out; condone some doubling / feathering–deleted work sometimes still shows up in rm assessor; must not be ruled; condone fairly straight with clear attempt at curve at minimum; be reasonably generous on attempt at symmetry e.g. condone minimum on y-axis for this mark
		ii	intersecting x-axis at 5 and −1 or ft from (i) and y-axis at −5	1		
		ii	turning point (2, −9)	1	seen on graph or identified as tp elsewhere in this part	may be implied by 2 and −9 marked on axes ‘opposite’ turning point **Examiner's Comments** Most candidates managed to solve the equation. Quite a number of candidates multiplied out the brackets and rearranged to form a quadratic equation in the traditional form. This was then usually solved by factorisation, but occasionally using the formula. Those who used the given form and took the square root of both sides were more inclined to find just one root, by ignoring the

						possibility that the square root of 9 could be −3. The quality of the parabolas varied enormously, but most candidates determined the coordinates of the turning point and made a good attempt. Some candidates did not consider the turning point and often had skewed parabolas with a minimum on the y-axis. A few candidates sketched cubics and received no marks.
			Total	5		
12		a	$(x + 2)^2 + 3$	B1(AO1.2) B1(AO1.1) [2]	For $b = 2$ For $c = 3$ or FT their b	
		b	Since $(x + b)^2 \geq 0$, the minimum value [or minimum point on the curve] occurs when the expression in the bracket is zero	E1(AO2.2a) [1]		
			Total	3		
13		a	Translation $\frac{b}{3a}$ parallel to x-axis	B1(AO1.1a) B1(AO1.1) [2]	oe	

		b	$\alpha - \dfrac{b}{3a}$	B1(AO2.2a) [1]		Graphs (Yr. 1)
			Total	3		